

# IFKSA-ESPRIT - Estimating the Direction of Arrival under the Element Failures in a Uniform Linear Antenna Array

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**Abstract**— This paper presents the use of Inverse Free Krylov Subspace Algorithm (IFKSA) with Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT) for the Direction-of-Arrival (DOA) estimation under element failure in a Uniform Linear Antenna Array (ULA). Failure of a few elements results in sparse signal space. IFKSA algorithm is an iterative algorithm to find the dominant eigenvalues and is applied for decomposition of sparse signal space, into signal subspace and the noise subspace. The ESPRIT is later used to estimate the DOAs. The performance of the algorithm is evaluated for various elements failure scenarios and noise levels, and the results are compared with ESPRIT and Cramer Rao Lower bound (CRLB). The results indicate a better performance of the IFKSA-ESPRIT based DOA estimation scheme under different antenna failure scenarios, and noise levels.

**Index Term s**— Inverse Free Krylov Subspace, ESPRIT, Direction-of-Arrival.

## I. INTRODUCTION

Antenna Array Signal Processing and estimation of Direction-of-Arrival of the signals impinging on the array of sensors or antennas, is a major functional requirement in radar, sonar and wireless radio communication systems. Generally, a Uniform Linear Antenna array (ULA) of a few tens to a few hundred elements (large antenna array) [1] are used for processing the signals impinging on the antenna array and estimate the DOAs. Among the various high resolution methods for DOA estimation, subspace based methods are most popular and powerful method. The popularity is due to its strong mathematical model to illustrate the underlying data model and it can with stand the perturbations in the data [2]. Subspace methods for DOA estimation searches for the steering vector associated with the directions of the signals of interest that are orthogonal to the noise subspace and are contained in the signal subspace. Once the signal subspace is extracted the DOAs are estimated. The decomposition is performed using the Eigen Value Decomposition (EVD) of the estimated received signal correlation matrix. The Multiple Signal Classification (MUSIC) [3] and Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT) [4] are the popular subspace based DOA estimation algorithms.

MUSIC algorithm is a spectral search algorithm and requires the knowledge of the array manifold for stringent array calibration requirement. This is normally an expensive and time consuming task. Furthermore, the spectral based methods require exhaustive search through the steering vector to find the location of the power spectral peaks and estimate the DOAs. ESPRIT overcomes these problems by exploiting the shift invariance property of the array. The algorithm reduces computational and the storage requirement. Unlike MUSIC, ESPRIT does not require the knowledge of the array manifold for stringent array calibration. There are number of variants and modification of ESPRIT algorithm [5][6][7][8][9]. ESPRIT algorithm is also extended for sparse linear antenna arrays or non-linear antenna arrays [10][11]. For nonlinear arrays the aperture extension and disambiguation is achieved by configuring the array geometry as dual size spatial invariance array geometry [10] or by representing the array as Virtual ULA, and using the Expectation-Maximization algorithm [11]. The subspace algorithms are heavily dependent on the structure of the correlation matrix and are unsuitable to handle sensor failures.

For handling sensor failure many modifications are proposed. Larson and Stocia [12], proposed a technique for estimating the correlation matrix of the incomplete data using the ML approach and shown improvement in MUSIC for handling sensor failure. However, it increases the complexity. A method for DOA estimator to handle sensor failure based on neural network is proposed by Vigneshwaran et al [13]. The technique can handle correlated signal sources, avoids the eigen decomposition. The drawback with these techniques is initialization of the network, and is performed by trial and error method. The authors in [15] proposed a DOA estimation technique by combining the EM algorithm with the MP method. The EM algorithm, expects the missing data and maximization is performance using the MP method. The method suffers for the drawback of increased complexity and requires good initialization. In [20], authors have proposed a Direct Data Domain DOA estimation algorithm, using the matrix completion procedure and Matrix Pencil (MP) method to first estimate the missing data observations and then estimating the DOA. However, the MP method is highly sensitive to the perturbations. Modification of ESPRIT DOA estimator to handle sensor failures is not present.

Decomposing the sparse correlation matrix is required to make the ESPRIT able to estimate the DOA for a faulty or incomplete data resulted from the failure of sensors.

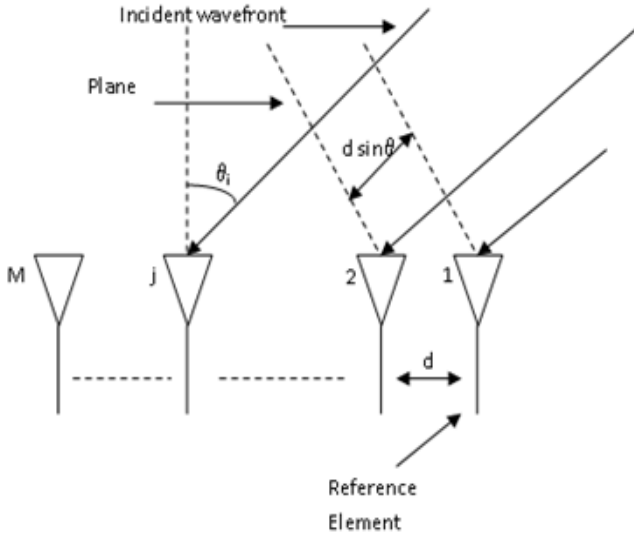


Figure 1: Illustration of ULA for DOA estimation

The very popular technique to solve the eigenvalue problem of a large sparse matrix is known as Krylov Subspace based techniques [15]. The reason for its popularity is due to its generality, simplicity and storage requirements. The iterative techniques of Krylov Subspace based techniques include Arnoldi approach, Lanczos algorithm, Jacobi-Davidson algorithm and Inverse Free Krylov Subspace Algorithms (IFKSA) proposed by Qin and Golub [16]. The study and analysis of the latter three techniques can be found in [15] [17]. The Krylov Subspace methods are applied in wide areas of application. For example, the method is used in control systems for estimating the parameters of a large sparse matrix, electric networks, segmentation in image processing and many others. IFKSA iteratively improves the approximate eigen pair, using either Lanczos or the Arnoldi iterations at each step through Rayleigh-Ritz projection procedure [17]. The algorithm is a very attractive due to the following reasons; first, the technique can be used to find any number of smallest eigenvalues (Largest can also be calculated), and second, the algorithm is less sensitive to perturbations.

The main objective of this paper is to extend conventional ESPRIT algorithm to handle the element failures. The technique uses IFKSA method to decompose the estimated correlation matrix into signal subspace which corresponds for larger eigenvalues and using the signal subspace the conventional ESPRIT is followed, resulting in IFKSA-ESPRIT, a robust DOA estimation algorithm to handle sensor failures.

Before evaluating the IFKSA-ESPRIT algorithm for antenna failures, we first carried out a detailed evaluation of the performance of IFKSA-ESPRIT DOA estimator by comparing with the conventional ESPRIT under different Signal-to-Noise Ratio (SNR) conditions and compared with Cramer Rao Lower Bound (CRLB) [18]. Next, the performance of IFKSA-ESPRIT scheme has been evaluated under different

array sensor failure scenarios and noise levels and the results are compared with CRLB. Finally, evaluating the performance of algorithm for large antennas, wide and narrow angles of arrival for various element failure scenarios and noise levels is performed. Results indicate that proposed technique has smaller errors for all the scenarios studied.

The remainder of the paper is organized as follows. In the following section the signal model is discussed, followed by overview for Krylov subspace in section 3. In Section 4, we discussed the proposed IFKSA-ESPRIT algorithm followed by the simulation results in section 5. Finally, conclusions are discussed in section 6.

## II. SIGNAL MODEL

The DOA estimation problem is to estimate the directions of plane wave incident on the antenna array. The problem can be looked as parameter estimation. We here mainly introduce the model of a DOA estimator. Consider an  $M$ -element uniformly spaced linear array. The array elements are equally spaced by a distance  $d$ , and a plane wave arrives at the array from a direction  $\theta$  off the array broadside. The angle is called the direction-of-arrival (DOA) or angle-of-arrival (AOA) of the received signal, and is measured clockwise from the broadside of the array.

Let  $N$  narrowband signals all centered around a known frequency, impinging on the array with a DOA,  $\theta$ . The received signal at the array is a superposition of all the impinging signal and noise. Therefore, the input data vector may be expressed as

$$\mathbf{x}(t) = \sum_{i=1}^N \mathbf{a}(\theta_i) s_i(t) + \mathbf{n}(t) \quad (1)$$

Where,

$$\mathbf{a}(\theta_i) = \left[ 1, e^{-j\frac{2\pi}{\lambda} d \sin \theta}, \dots, e^{-j\frac{2\pi}{\lambda} d (M-1) \sin \theta} \right]^T \quad (2)$$

$\mathbf{a}(\theta_i)$  is the steering vector of the antenna array. Here,  $T$  represents the complex conjugate transpose.

$$\mathbf{X} = \mathbf{A}(\boldsymbol{\theta}) \mathbf{s} + \mathbf{W} \quad (3)$$

Where,  $\mathbf{X}$  is the array output matrix of size  $M \times K$ ,  $\mathbf{A}$  is the complete steering matrix of size  $M \times N$  function of the DOA vector  $\boldsymbol{\theta}$ ,  $\mathbf{s}$  is signal vector of size  $N \times K$  and  $\mathbf{W}$  is the noise vector of size  $M \times K$ . Here,  $K$  is the number of snapshots. Eq. (3) represents the most commonly used narrowband input data model. When an element fails there will be output from the failed element. The received sparse signal vector due to failure of elements is

$$\mathbf{X}' = \mathbf{A}'(\boldsymbol{\theta}) \mathbf{s} + \mathbf{W}' \quad (4)$$

Where,  $M'$  is number of elements functioning. The correlation matrix of the sparse received signal is

$$\mathbf{R}_{\mathbf{x}'\mathbf{x}'} = \frac{1}{K} \sum_{k=1}^K \mathbf{x}_k' \mathbf{x}_k \quad (5)$$

The objective of the papers is to estimate the DOA means to find out the value of the parameter  $\theta_i$ , when a few of the elements fail to work.

### III. OVERVIEW OF KRYLOV SUBSPACE METHOD

We use Krylov subspace method [15] [17] to decompose the received signal space into signal subspace and noise

TABLE I. INVERSE FREE PRECONDITIONED KRYLOV SUBSPACE - ESPRIT ALGORITHM (IFKS-ESPRIT)

Input $R'$ , Number of Signal Sources	
Step 1:	Compute the Eigen decomposition of the array covariance matrix using the Inverse Free preconditioned Krylov subspace (IFKS) algorithm
<i>IFKS Algorithm</i> ( ):	
Step a:	Set $m \geq 1$ , $\Phi \leftarrow R'$ and an initial approximation $\mathbf{u}_0$ with $\ \mathbf{u}_0\  = 1$
Step b:	For $k = 0, 1, \dots$ , until Convergence Construct a basis $\mathbf{V}_m = [\mathbf{v}_0, \mathbf{v}_1, \dots, \mathbf{v}_m]$ for the subspace $\mathcal{K}^n = \text{span}(\mathbf{u}_k, (\Phi - \rho_k)\mathbf{u}_k, (\Phi - \rho_k)^2\mathbf{u}_k, \dots, (\Phi - \rho_k)^{n-1}\mathbf{u}_k)$
Step c:	Form $\Phi_m = \mathbf{V}_m^T (\Phi - \rho_k) \mathbf{V}_m$
Step d:	Compute the $(\lambda_i, \mathbf{y}_i)$ eigenpairs of $\Phi_m$ , and select the desired ones. $\rho_{k+1} = \rho_k + \lambda_1 \quad \mathbf{u}_{k+1} = \mathbf{V}_m \mathbf{y}_1$
End;	
Step 2:	Estimate the DOA using ESPRIT
<i>ESPRIT</i> ( ):	
Step a:	Form the signal subspace $\mathbf{E}$ using the eigenvectors obtained from step 1
Step b:	Form the matrices $\mathbf{E}_0$ and $\mathbf{E}_1$
Step c:	Estimate the DOA solving $[\mathbf{E}_0, \mathbf{E}_1]$ .
End;	

subspace spanned by the dominant eigenpairs and the smaller eigenpairs respectively. The decomposing is the first step in ESPRIT DOA estimator. A Krylov subspace of dimension  $n$  generated by a vector  $\mathbf{u}$  and the matrix  $\Phi$  is,

$$\mathcal{K}^n(\mathbf{u}, \Phi) = \text{span}(\mathbf{u}, \Phi\mathbf{u}, \Phi^2\mathbf{u}, \dots, \Phi^{n-1}\mathbf{u}) \quad (6)$$

Where,  $\mathbf{u}$  is the initial eigenvector. The idea is to generate an orthonormal basis  $\mathbf{V} = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\}$ , of  $\mathcal{K}^n$  and seek an approximation solution to the original problem from the subspace. Since we seek approximate eigenvector  $\tilde{\mathbf{u}}, \tilde{\mathbf{u}} \in \mathcal{K}$ , we can write

$$\tilde{\mathbf{u}} = \mathbf{V}\mathbf{y} \quad (7)$$

The approximate eigenpair  $(\tilde{\lambda}, \tilde{\mathbf{u}})$  is obtained by imposing Galerkin condition [17]

$$\mathbf{v}_j^H (\Phi \mathbf{V}\mathbf{y} - \tilde{\lambda} \mathbf{V}\mathbf{y}) = 0, \quad \mathbf{v}_j \in \mathcal{K} \quad j = 1, 2, \dots, N \quad (8)$$

Or

$$(\Phi - \tilde{\lambda} \mathbf{I}) \tilde{\mathbf{u}} \perp \mathcal{K}^n \quad (9)$$

Therefore  $\mathbf{y}$  and  $\tilde{\lambda}$  must satisfy

$$\mathbf{B}_n \mathbf{y} = \tilde{\lambda} \mathbf{y} \quad (10)$$

Where  $\mathbf{B}_n = \mathbf{V}^H \Phi \mathbf{V}$ .

The approximate eigenvalues  $\tilde{\lambda}$  resulting from the projection

process are all the eigenvalues of the matrix  $\mathbf{B}_n$ . The associated eigenvectors are the vectors  $\mathbf{V}\mathbf{y}$  in which  $\mathbf{y}$  is an eigenvector of  $\mathbf{B}_n$  associated with the  $\tilde{\lambda}$ . The procedure for numerically computing the Galerkin condition to eigenvectors and eigenvalues of  $\Phi$  is known as Rayleigh-Ritz procedure [15]. The approximation is updated convergence. There are many algorithms used to solve the above problem, namely Arnoldi algorithm, Lanczos algorithm and Jacobi-Davidson algorithm. Golub and Yi [8] proposed another algorithm known Inverse Free Krylov Subspace Algorithm. Inverse Free Krylov Subspace method starts with the initial approximation and aims to improve the approximation by minimizing the Rayleigh Quotient on that subspace.

### IV. IFKSA-ESPRIT ALGORITHM

The algorithm for estimating the DOA from a faulty ULA is given in Table 1. It consists of two steps. Step 1 is decomposing the correlation matrix for signal subspace and the noise subspace using IFKSA technique. In the second step, the conventional ESPRIT algorithm is applied to estimate the DOAs. The algorithm starts with an initial approximation  $(\rho_0, \mathbf{u}_0)$  and aims at improving through Rayleigh-Ritz projection [16] on a certain subspace, i.e., by minimizing the Rayleigh quotient on that subspace

$$\rho(\mathbf{u}) = \frac{\mathbf{u}^T \Phi \mathbf{u}}{\mathbf{u}^T \mathbf{u}} \quad (11)$$

The Rayleigh quotients are important both for theoretical and practical purposes. The set of all possible Rayleigh quotients as  $\mathbf{u}$  runs over  $\mathbb{C}$  is called the field values of  $\Phi$ . The gradient of Rayleigh quotient at  $\mathbf{u}_0$  is

$$\begin{aligned} \nabla \rho(\mathbf{u}_0) &= \rho_0 = \nabla \left( \frac{\mathbf{u}_0^T \Phi \mathbf{u}_0}{\mathbf{u}_0^T \mathbf{u}_0} \right) \\ &= \frac{\partial}{\partial \mathbf{u}_i} \left( \frac{\mathbf{u}_0^T \Phi \mathbf{u}_0}{\mathbf{u}_0^T \mathbf{u}_0} \right) \\ &= \frac{\frac{\partial}{\partial \mathbf{u}_i} (\mathbf{u}_0^T \Phi \mathbf{u}_0) \mathbf{u}_0^T \mathbf{u}_0 - \mathbf{u}_0^T \Phi \mathbf{u}_0 \frac{\partial}{\partial \mathbf{u}_i} \mathbf{u}_0^T \mathbf{u}_0}{(\mathbf{u}_0^T \mathbf{u}_0)^2} \\ &= \frac{2((\Phi \mathbf{u}_0) - \rho_0 \mathbf{u}_0)}{\mathbf{u}_0^T \mathbf{u}_0} \end{aligned} \quad (12)$$

The well known steepest descent method chooses a new approximate eigenvector  $\mathbf{u}_1 \in \text{span}\{\mathbf{u}_0, \rho_0\}$  by minimizing  $\rho(\mathbf{u}_1)$ . Clearly, this is a Rayleigh-Ritz projection method on the subspace  $\mathcal{K}^n = \text{span}\{\mathbf{u}_0, (\Phi - \rho_0)\mathbf{u}_0\}$ . The new approximation  $\mathbf{u}_1$  is constructed by inexact inverse iteration [9]

$$\mathbf{u}_1 = (\Phi - \rho_0)^{-1} \mathbf{u}_0 \quad (13)$$

then  $\mathbf{u}_1$  is indeed chosen from a Krylov subspace as generated by  $(\Phi - \rho_0)$ . Therefore, a new approximate eigenvector is found from the Krylov subspace of some fixed  $n$

$$\mathcal{K}^n = \text{span} \left( \mathbf{u}_0, (\Phi - \rho_0) \mathbf{u}_0, (\Phi - \rho_0)^2 \mathbf{u}_0, \dots, (\Phi - \rho_0)^{n-1} \mathbf{u}_0 \right) \quad (14)$$

by using the Rayleigh-Ritz projection method. The improved approximate eigenvector is obtained by iterating the above steps. The step 2 of the algorithms is the conventional ESPRIT algorithm. ESPRIT [4] is a computationally efficient of DOA estimation. In this technique two sub-arrays are formed with the identical displacement vector, that is, in same distance and same direction relative to first element. The array geometry should be such that the elements could be selected to have this property. The ESPRIT

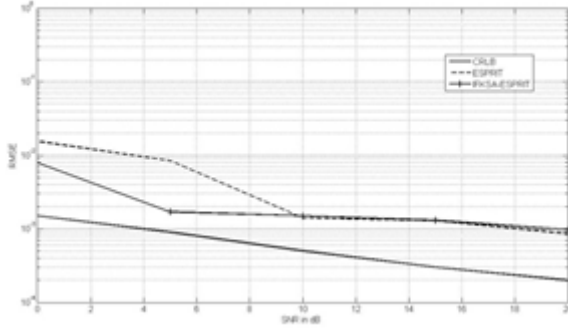


Fig. 1: RMSE vs SNR plot of CRLB, ESPRIT and IFKSA-ESPRIT algorithms for  $M = 11$  and all elements are working

algorithm uses the structure of the ULA steering vectors in a slightly different way. The observation here is that  $\mathbf{A}$  has a so called shift structure. Define the sub-matrices,  $\mathbf{A}_0$  and  $\mathbf{A}_1$  by deleting the first and last columns from respectively, and note that and are related as

$$\mathbf{A}_1 = \mathbf{A}_0 \mathbf{\Omega} \quad (15)$$

Where,  $\mathbf{\Omega}$  is a diagonal matrix having the “roots”  $e^{j\theta_i}$ ,  $i = 1, 2, \dots, N$ . Thus DOA estimation problem can be reduced to that of finding .

Let and are the two matrices obtained by deleting the first column and last column of (5). Also let and are the two matrices with their columns denoting the eigenvectors corresponding to the largest eigenvalues of the two autocorrelation matrixes and respectively. As these two sets of eigenvectors span the same - dimensional signal space, it follows that these two matrixes and are related by a unique nonsingular transformation matrix , that is

$$\mathbf{E}_1 = \mathbf{E}_0 \mathbf{\Psi} \quad (16)$$

These two matrices  $\mathbf{E}_0$  and  $\mathbf{E}_1$  are related to the steering vector  $\mathbf{A}_0$  and  $\mathbf{A}_1$  by another unique nonsingular transformation matrix  $\mathbf{\Gamma}$ , as the same signal subspace is spanned by these steering vectors. Thus

$$\begin{aligned} \mathbf{V}_0 &= \mathbf{A}_0 \mathbf{\Gamma} \\ \mathbf{V}_1 &= \mathbf{A}_1 \mathbf{\Gamma} = \mathbf{A}_0 \mathbf{\Omega} \mathbf{\Gamma} \end{aligned} \quad (17)$$

Substituting for  $\mathbf{V}_0$  and  $\mathbf{V}_1$ , one obtains

$$\mathbf{\Gamma} \mathbf{\Psi} \mathbf{\Gamma}^{-1} = \mathbf{\Omega} \quad (18)$$

Which states that the eigenvalues of  $\mathbf{\Psi}$  are equal to the diagonal elements of  $\mathbf{\Omega}$  and that the columns of  $\mathbf{\Gamma}$  are eigenvectors of . This is the main relationship in the development of ESPRIT.

#### IV. SIMULATION RESULTS

In this section we examine the performance of the proposed algorithm for estimating the DOAs, when a few elements fail to work. We consider a ULA of  $M$  elements, with inter-element spacing of . The 300 snapshots of the signals, which are assumed as complex exponential sequences

$$s(t) = e^{j\phi(t)} \quad t = 1, 2, \dots, 300 \quad (19)$$

is considered for the simulation. The Signal-to-Noise Ratio (SNR) is defined as

$$SNR = 10 \log_{10} \frac{\sigma_s^2}{\sigma_w^2} \quad (20)$$

Where,  $\sigma_s^2$  and  $\sigma_w^2$  are signal power and noise power respectively. The Root Mean Square Error (RMSE) is used to evaluate the performance of the algorithm.

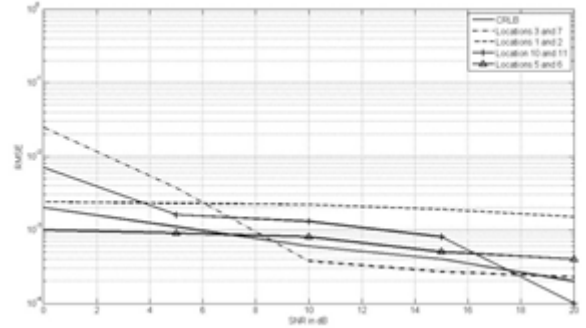


Fig. 2: RMSE vs SNR plot for estimation of DOA for various location of failure of two elements by IFKSA-ESPRIT algorithm for  $M = 11$  elements

The RMSE is defined as

$$RMSE = \sqrt{E \{ [\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}]^2 \}} \quad (21)$$

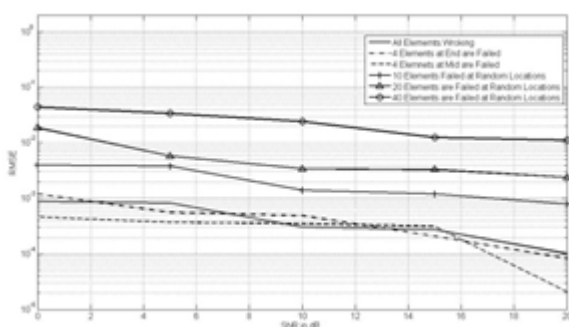
Where,  $E(\cdot)$  is the mean value,  $\boldsymbol{\theta}$  is the actual DOAs vector and  $\hat{\boldsymbol{\theta}}$  is the vector of estimated DOAs. We considered three simulation examples.

First, we evaluate the proposed method for the case when all the elements are functioning. The results are compared with the conventional ESPRIT and Cramer-Rao Lower Bound (CRLB). The SNR vs RMSE plot is plotted for evaluation of the algorithms. Two signals of equal amplitudes are assumed to be impinging in the array of size 11 elements for the DOAs  $[5^\circ -5^\circ]$ . It can be observed from Fig. (1), the algorithm is showing improved performance when compared to ESPRIT at low SNR values and also able to achieve the CRLB [10].

In our second example we evaluate only the proposed method. The CRLB is calculated for the failed elements. We assume to elements are failed at some fixed positions for calculating the CRLB for various SNR values. The proposed

algorithm is evaluated for failure of two elements at various locations. For example, the locations we selected are (3<sup>rd</sup> and 7<sup>th</sup>) elements at the ends of the array, (1<sup>st</sup> and 2<sup>nd</sup>) first two elements, (10<sup>th</sup> and 11<sup>th</sup>) last two elements, and (5<sup>th</sup> and 6<sup>th</sup>) middle two elements. With the failure of two elements the algorithm is able to estimate the DOAs and achieve CRLB, shown in Fig. (2). Furthermore, when the (3<sup>rd</sup> and 7<sup>th</sup>) elements fail, at low SNR values the algorithm shows performance degradation.

In our final example, we consider a large array of size 100 elements. The number of signals impinging on the array is taken as 10 of equal amplitudes. The actual DOAs of the signals are assumed as  $[-30^\circ, -20^\circ, -10^\circ, -5^\circ, 0^\circ, 5^\circ, 10^\circ, 20^\circ, 30^\circ, 40^\circ]$ . The algorithm performance for this large array is evaluated for 6 cases; all the 100 elements are functioning, 4 elements are failed at the end i.e, at locations 97, 98, 99 and 100, 4 elements are failed at the beginning i.e, at locations 1, 2, 3 and 4, 10 elements at random locations, 20 elements at random locations and finally 40 elements at random locations. The SNR vs RMSE plot is plotted in Fig. (3). It is observed that except the last case, of failure of 40 elements at random locations, the proposed algorithm is able to estimate the DOAs. When 40 elements fail to work the performance gets worse.



#### IV. CONCLUSION

In this paper, the performance of the IFKSA-ESPRIT DOA estimator is evaluated for various antenna array element failures in noisy environment. Results indicate that the algorithm performance better than the conventional ESPRIT at low SNR values, when all the elements are functioning. For failure case, IFKSA-ESPRIT is able to estimate the DOAs. However, when there is large number of element failures the algorithm is failed to estimate the DOA, which need further investigation.

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